RADIATIVE HEAT TRANSFER IN A FLAT NONISOTHERMAL BED

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A general formula is derived for the radiation flux impinging on the surface of the bed. The problem of self-radiation of a non-thick nonisothermal bed is solved, as well as the problem of self-radiation of a semi-infinite medium whose temperature remains constant everywhere except for a non-thick boundary layer.

1. We consider the problem of radiative heat transfer in a planar bed (layer) of an absorbing and scattering medium. The temperature field in the medium will be assumed known. We assume that the effective radiation from the surfaces bounding the bed is hemispherical, and that the temperature varies along the axis perpendicular to the surface layers. We assume the medium and walls gray, since otherwise the problem would become complicated as we attempt to integrate over the entire wavelength spectrum. The problem as posed has been solved [1] by the Eddington method. In some cases, the accuracy of the results reported there [1] is judged inadequate. For example, the emissivity factor of a semi-infinite nonscattering layer is reported to be $4/(2+\sqrt{3})$, instead of the correct value $\varepsilon = 1$.

Since the effective radiation from surfaces bounding the layer is hemispherical, we can resort to a method similar to the one advanced by O. E. Vlasov [2], in order to find the radiation flux. In the case of radiation flux impinging on the surface $\tau = 0$ on the side of the process medium, we have

$$q(0) = q_h(0) + [r_2 q(\tau_0) + q_2]D + [q(0)r_1 + q_1]R,$$

and for radiation flux impinging on the second bounding plane $\tau = \tau_0$ from the side of the process medium, we have

$$q(\tau_0) = q_h(\tau_0) + [q(0)r_1 + q_1]D + [q(\tau_0)r_2 + q_2]R.$$

These equations give the law of energy conservation in this application. Solving them jointly, we have

$$q(0) = \frac{(1 - r_2 R)[q_h(0) + q_2 D + q_1 R] + r_2 D[q_h(\tau_0) + q_1 D + q_2 R]}{(1 - r_1 R)(1 - r_2 R) - D^2 r_1 r_2}. \tag{1}$$

The flux of radiation impinging on the $\tau = \tau_0$ surface is found from Eq. (1) by interchanging subscripts $1 \rightleftharpoons 2$ and $0 \rightleftharpoons \tau_0$. In the case where $q_h(0) = q_h(\tau_0)$, Eq. (1) goes over into a more familiar equation [3]. When $q(0)$ and $q(\tau_0)$ are known, the problem of radiative heat transfer in the plane layer becomes completely solvable. There is a considerable body of literature devoted to calculations of the variables R and D. An approximate solution of the problem of the flux of self-radiation of the layer is presented below.

2. The radiative transfer equation for intensity I integrated over the azimuthal angle is written, for a plane bed, in the form [5]

$$\mu \frac{\partial I}{\partial \tau} + I = \frac{\lambda}{2}\int_{-1}^{1} P(\mu, \mu') I(\tau, \mu') d\mu' + j(\tau). \tag{2}$$

The function j is expressed in terms of the temperature of the surroundings as follows:

$$j(\tau) = 2(1 - \lambda)\sigma T^4(\tau). \tag{3}$$

Since we are required to find the flux of self-radiation of the bed, the boundary conditions for Eq. (2) are zero conditions:

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TABLE 1. Ratio of Luminosity of Layer to σT_0^4

λ	$\alpha=2$			$\alpha=1$		$\alpha=0,5$		
	$q_h(0)/\sigma T_0^4$	$q_h(1)/\sigma T_0^4$	$q_\infty/\sigma T_0^4$	$\varepsilon(1)$	$\varepsilon(\infty)$	$q_h(0)/\sigma T_0^4$	$q_h(1)/\sigma T_0^4$	$q_\infty/\sigma T_0^4$
0	8,36	4,89	8,58	0,781	1,000	0,305	0,522	0,525
0,2	7,35	4,68	7,65	0,711	0,958	0,294	0,460	0,538
0,4	6,19	4,30	6,59	0,617	0,890	0,268	0,385	0,536
0,6	4,66	3,57	5,18	0,486	0,799	0,223	0,291	0,523

$$I(0,\ \mu>0)=0;\quad I(\tau_0,\ \mu<0)=0, \tag{4}$$

where $\tau_0 = \int_0^L Kdx$ is the optical thickness of the layer.

Once the intensity is known, there is little difficulty in finding the self-radiation flux of the layer

$$q_h(\tau_0)=\int_0^1 \mu I(\tau_0,\ \mu)\,d\mu,$$
$$q_h(0)=-\int_{-1}^0 \mu I(0,\ \mu)\,d\mu. \tag{5}$$

The intensity of radiation from an elemental volume of the heated medium is proportional to the absorptivity. The closer the elemental volume is located to the boundary of the medium, the greater the portion of its radiation that will find its way outside. But scattering need not be taken into account at small optical thicknesses. Consequently, we need not take scattering into account either in problems dealing with self-radiation of the medium when the optical thicknesses of the medium are not too great, and instead we can consider only the absorption of the radiation. Neglecting scattering as a factor is of course equivalent to assuming the scattering indicatrix to be infinitely extended in the forward direction. Accordingly, the more the scattering indicatrix is extended forward, the more accurate such an approximation would be. The radiative transfer equation in that approximation becomes

$$\mu\frac{\partial I}{\partial\tau}+(1-\lambda)\,I=j(\tau). \tag{6}$$

The solution of this equation, with the boundary conditions (4), appears in the form

$$I(\tau,\ \mu>0)=\frac{\exp\left[-(1-\lambda)\frac{\tau}{\mu}\right]}{\mu}\int_0^\tau j(\tau')\exp\left[(1-\lambda)\frac{\tau'}{\mu}\right]d\tau',$$
$$I(\tau,\ \mu<0)=\frac{\exp\left[-(1-\lambda)\frac{\tau}{\mu}\right]}{\mu}\int_{\tau_0}^\tau j(\tau')\exp\left[(1-\lambda)\frac{\tau'}{\mu}\right]d\tau'. \tag{7}$$

For the flux of intrinsic radiation from the layer we have, from Eq. (5)

$$q_h(\tau_0)=\int_0^{\tau_0} j(\tau')\,E_2\,[(1-\lambda)\,(\tau_0-\tau')]\,d\tau',$$
$$q_h(0)=\int_0^{\tau_0} j(\tau')\,E_2\,[(1-\lambda)\,\tau']\,d\tau'. \tag{8}$$

Here E_2 is the second-order integral exponential function. The nth-order integral exponential function is given by the equation

$$E_n(x)=\int_0^1 \exp\left[-\frac{x}{\mu}\right]\mu^{n-2}\,d\mu.$$

If the function j is a second-degree polynomial in τ,

$$j(\tau) = a_0 + a_1\tau + a_2\tau^2, \tag{9}$$

then we readily obtain, from Eqs. (8),

$$q_h(0) = \frac{a_0}{1-\lambda} A(y) + \frac{a_1}{(1-\lambda)^2} B(y) + \frac{a_2}{(1-\lambda)^3} C(y), \tag{10}$$

$$q_h(\tau_0) = \frac{a_0}{1-\lambda} A(y) + \frac{a_1}{(1-\lambda)^2} [yA(y) - B(y)] + \frac{a_2}{(1-\lambda)^3} [y^2 A(y) - 2yB(y) + C(y)], \tag{11}$$

where

$$\begin{gathered} y = (1-\lambda)\tau_0; \quad A(y) = \frac{1}{2} - E_3(y); \\ B(y) = \frac{1}{3} - yE_3(y) - E_4(y); \\ C(y) = \frac{1}{2} - y^2E_3(y) - 2yE_4(y) - 2E_5(y). \end{gathered} \tag{12}$$

If the layer produces no scattering, then the solution will be an exact one.

For the emissivity factor of the isothermal layer in this approximation, we find

$$\varepsilon = 1 - 2E_3[(1-\lambda)\tau_0]. \tag{13}$$

Comparison of results based on this formula and more exact data for a spherical scattering indicatrix [4, 6] show satisfactory accuracy at $\tau_0 \le 1.5$.

3. We consider now the radiation from a semi-infinite medium bounded by the $\tau = 0$ plane. We shall assume that the temperature of the semi-infinite medium is constant everywhere, except for the boundary layer, the optical thickness τ^* of which is not high. When calculating the flux of intrinsic radiation from the boundary layer, we may ignore scattering, on the basis of the above. To calculate the flux of intrinsic radiation from the semi-infinite layer, the medium beyond the boundary layer is replaced by an equivalent surface of reflectivity R_∞ and by a flux of intrinsic radiation q_∞^*. We assume, in an approximation, that both the radiation impinging on that surface and the effective radiation from the equivalent surface are hemispherical. In that case we obtain, for the flux of intrinsic radiation from the semi-infinite medium, and using Eq. (1),

$$q_\infty = q^*(0) + D(\tau^*) \frac{q_\infty^* + R_\infty q^*(\tau^*)}{1 - R_\infty R(\tau^*)}, \tag{14}$$

where $q^*(0)$ is the flux of intrinsic radiation from the boundary layer on the boundary of the medium $\tau = 0$; $q^*(\tau^*)$ is the flux of intrinsic radiation from the boundary layer on the boundary of the medium τ^*. The data for the reflectivity R_∞ of the semi-infinite medium can be found in [7] for a spherical scattering indicatrix and for the simplest scattering indicatrix extended in the forward direction. More complete data relevant to a spherical scattering indicatrix are found in [4]. A check reveals excellent agreement between Eq. (14) for an isothermal semi-infinite layer with a spherical scattering indicatrix.

The method for replacing the medium beyond the boundary layer with an equivalent surface was employed in [8].

4. Consider an illustrative example of calculations based on the procedure set forth. Let

$$j = a_0 + a_1\tau. \tag{15}$$

We then introduce the notation

$$\begin{gathered} a_0 = 2(1-\lambda)\sigma T_0^4 \tau^4, \\ a_1 = 2(1-\lambda)\sigma T_0^4 \frac{1-\alpha^4}{\tau_0}, \end{gathered} \tag{16}$$

i.e., we assume that the temperature of the medium is T_0 on the boundary $\tau = \tau_0$, and that the temperature of the medium on the boundary $\tau = 0$ is α times greater, so that the fourth power of the temperature is a linear function of the optical depth τ.

Table 1 lists the results of calculations of the ratio of the luminosity of a layer of optical thickness $\tau_0 = 1$ to the product σT_0^4 for different values of the ratio of the scattering coefficient to the attentuation coefficient, and for several α values. This table also lists results of calculations of the ratio of luminosity of the semi-infinite medium to the product σT_0^4. It is assumed in these calculations that the boundary layer has an optical density $\tau^* = 1$. The pattern of change in the temperature of the boundary layer is the same as in the preceding problem, i.e., Eqs. (15) and (16) are valid for the boundary layer. The temperature of the semi-infinite medium beyond the boundary layer was assumed constant and equal to T_0, and scattering as isotropic.

NOTATION

$q(0)$	is the radiative flux impinging on the surface $\tau = 0$ on the layer side;
$q(\tau_0)$	is the radiative flux impinging on the surface $\tau = \tau_0$ on the layer side;
$q_h(0)$, $q_h(\tau_0)$	are the intrinsic radiative fluxes on the surfaces $\tau = 0$ and $\tau = \tau_0$ respectively;
D	is the transmitting power of the layer;
r_1, r_2	are the reflectivities of the surfaces $\tau = 0$ and $\tau = \tau_0$ respectively;
q_1, q_2	are the fluxes of intrinsic radiation of the surfaces $\tau = 0$ and $\tau = \tau_0$ respectively;
R	is the reflecting power of the layer;
λ	is the ratio of the scattering coefficient to the attenuation coefficient;
K	is the attenuation coefficient;
τ_0	is the optical thickness of the layer;
τ	is the optical depth;
I	is the intensity of the radiation integrated over the azimuthal angle;
μ	is the cosine of the angle between the axis and the direction of the radiation;
σ	is the Stefan–Boltzmann constant;
T	is the absolute temperature;
E_n	is the integral nth-order exponential function;
τ^*	is the optical thickness of the boundary layer;
R_∞	is the reflecting power of the semi-infinite medium;
q_∞^*	is the flux of intrinsic radiation from the isothermal semi-infinite medium;
$q^*(0)$	is the flux of intrinsic radiation from the boundary layer on the boundary of the medium $\tau = 0$;
$q^*(\tau^*)$	is the flux of intrinsic radiation from the boundary layer on the boundary $\tau = \tau^*$;
q_∞	is the flux of intrinsic radiation of the semi-infinite medium;
ε	is the emissivity factor of the layer;
S	is the surface area.

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